A SPATIAL–TEMPORAL ANALYSIS OF THE DEVELOPMENT OF A LOG-SPIRAL SHAPED EMBAYMENT

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ABSTRACT

With the use of semi-annual data collected from May 1985 to May 1995, this study assesses the evolution of a log-spiral shaped embayment at the Northeast Beach, located at Canada's southernmost promontory. The emplacement of stone breakwaters initiated scour of the shoreline, and the embayment which developed thereafter adjusted to reflect changes in lake levels, and naturally occurring variance in wave and sediment dynamics. Each of the 21 recorded measurements of the embayment configuration is assessed for characteristics of log-spiral morphology. The results demonstrate that logarithmic-spiral curves fit the 21 recorded embayment configurations very closely. The significant parameters and statistics associated with each log-spiral curve show that no heteroschedacity of variance is present among the residuals and only one significant autocorrelation is observed. The appropriateness of the log-spiral fits is verified by the coefficients of determination which, with the exception of the November 1988 dataset, are all over the required value of 0·98.

Both the spiral angle and the radius show decreasing trends through time, and the graphical plots illustrate only minor fluctuations after May 1991. These results suggest that the embayment may have shifted from some form of quasi-equilibrium condition to approach a state of dynamic equilibrium. The persistence of the logarithmic-spiral configuration through time supports the findings of other investigators who claim that the log-spiral planform represents the equilibrium embayment shape that might be expected on the flank of a breakwater system. © 1997 by John Wiley & Sons, Ltd.

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INTRODUCTION

Ever since Yasso (1965) proposed the fit of the plan shape of headland-bay beaches to a logarithmic-spiral, researchers have observed log-spiral bays in coastal locations throughout the world. According to LeBlond (1979, p. 1099), 'the logarithmic-spiral shape observed in many headland-bay beaches may be understood in terms of a balance between the effects of the headland and the nearshore bathymetry on wave refraction and diffraction and a relation between beach slope, wave energy and grain size'. While it is accepted that the shorelines of some beaches which lie in the lee of a headland approximate the shape of a logarithmic-spiral, far more research remains to be done on the evolution of log-spiral bays in shoreline areas affected by artificial structures, for example, jetties, groynes, marinas, harbours and breakwaters.

The experimental study of Silvester (1970) produced a log-spiral in the central portion of artificial beaches. This study observed that the presence of stone breakwaters might initiate the development of embayments with logarithmic-spiral shapes, and concluded that the log-spiral is an equilibrium product of headland-bay processes as long as the direction of approaching waves remains fairly constant. Walton (1977) noted that logarithmic-spiral scour forms developed on the flanks of headlands or breakwaters, and claimed that the logarithmic-spiral form normally represents the equilibrium embayment shape that could develop on the flank of a breakwater system. According to McDougal *et al*. (1987), flanking erosion effects can be associated with shoreline scour on the downdrift side of long breakwaters. The findings from McDougal *et al*.'s study allowed the conclusion to be made that the depth of scour and the length of scour should be proportional to the length of the breakwater system.

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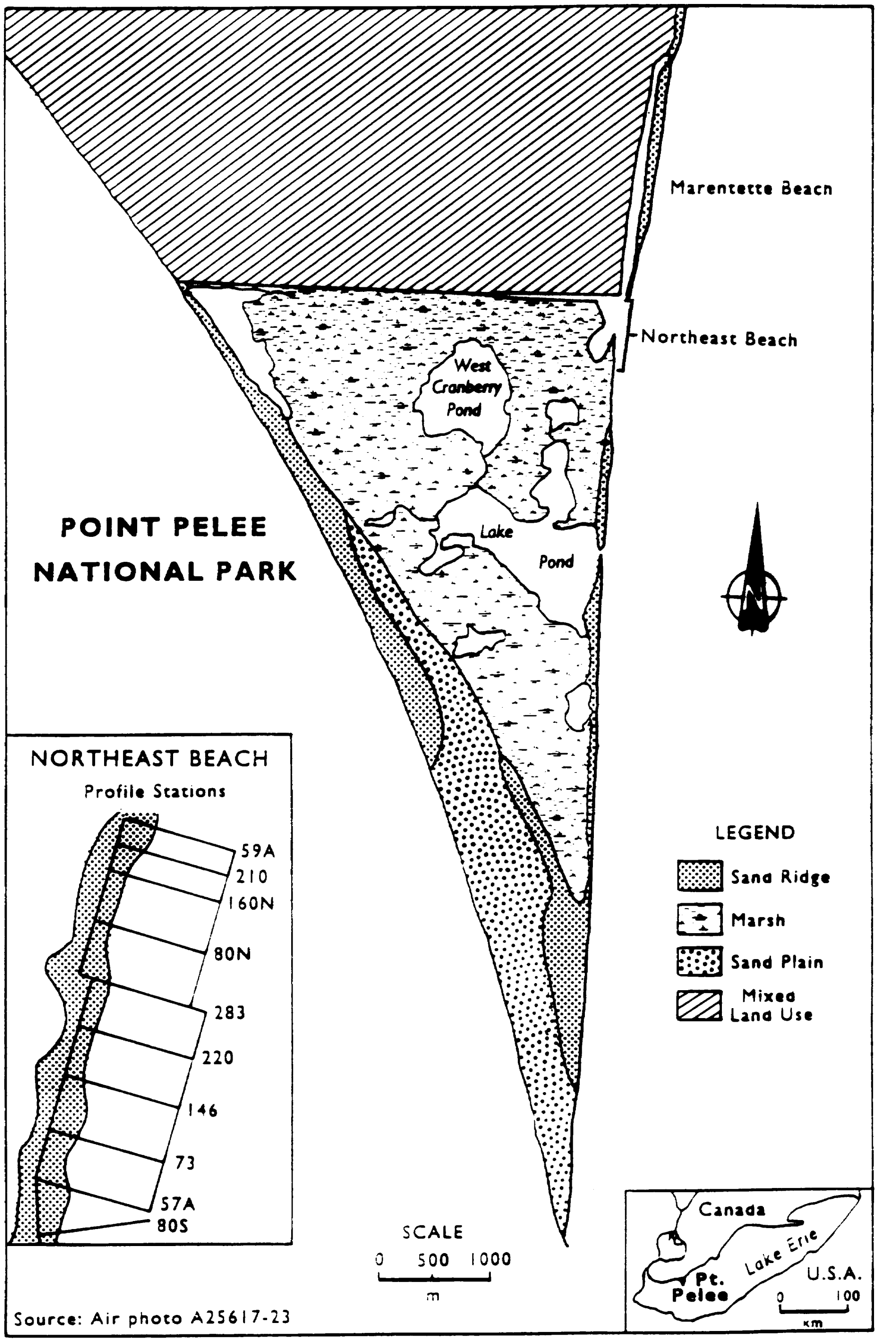


Figure 1. Location of the Northeast Beach, Point Pelee, Ontario, Canada (drawn from aerial photograph A25617-23, September 1980, obtained from the Department of Energy, Mines & Resources, Government of Canada).

The geometric trends in the evolution of a small log-spiral embayment near to a marina have been reported by Terpstra and Chrzastowski (1992), who found that the development of the embayment throughout the nine-month study approximated a log-spiral form. With the use of examples from two Japanese harbours, Hsu *et al.*

(1993) described how shoreline structures generate logarithmic-spiral shaped embayments. This study stated that the geomorphological concept of log-spiral shapes is useful in harbour planning.

Since the literature provides evidence that artificial structures can create conditions conducive to the development of log-spiral bays, this study will fit logarithmic-spiral curves to the embayment which developed near the armour stone breakwater emplaced in the vicinity of the Northeast Beach, Point Pelee, Canada (Figure 1). The results from this study will be beneficial to shoreline managers who have to implement decisions on the use of structural protection devices for shoreline protection purposes.

THE STUDY AREA

Point Pelee, the southernmost promontory of Canada, extends nearly 10 km into Lake Erie. Since Point Pelee is flanked by water on three sides, the shoreline is exposed to the variable effects of waves and longshore current flows. According to East (1976), the most effective wind-generated wave action associated with shoreline erosion comes from the east, because the fetches on the eastern flank of the Point are much greater than those from the west. A majority of the destructive storm-generated significant waves, however, emanate from the northeast (Kovacs, 1987). The movement of sediment through the shoreline of Point Pelee is controlled by persistent longshore current flows. Two main current systems flow south along each side of Point Pelee, while from the southwest a third surface current flows towards the tip of the Point. The Northeast Beach is influenced by longshore currents which flow south two-thirds of the time (Bukata *et al*., 1974).

Point Pelee has experienced significant alterations which interrupted the establishment of steady-state conditions. Point Pelee should have been moving towards a state of dynamic equilibrium where the rates of sediment inflow equal the rates of sediment outflow, leading to a steady-state morphology. However, the ill-conceived and uncoordinated shoreline management strategies at Point Pelee can be blamed for preventing the beaches at the Point from attaining dynamic equilibrium conditions. Groynes, jetties, tetrapods, breakwaters and other structures have been implemented at Point Pelee to control shoreline erosion, but the structures have impeded longshore sedimentary movements, and severely reduced the volumes of sediment flowing into the system (East, 1976; LaValle, 1990).

The Northeast Beach, the area of study in this paper, has been adversely affected by artificial structures. Beach erosion has been a problem of concern since 1973, when a major breach developed and threatened the ecological integrity of the interior marshlands. As a result, a system of concrete tetrapods was emplaced along the northern 300 m of the beach, and an artificial berm was constructed to fill the breach. In 1979, a sediment renourishment programme was instituted along the central portion of the Northeast Beach. Also, in 1984 a large armour stone breakwater was constructed along Marentette Beach immediately north of the Northeast Beach. Although this breakwater conflicted with the management requirements of Parks Canada it was, nevertheless, emplaced on a narrow sand bar base. The Northeast Beach study area includes 260 m of the Marentette Beach armour stone breakwater which is over 1000 m long, and nearly 2 m high.

In spite of the presence of a treble line of concrete tetrapods designed to reduce shoreline erosion at the Northeast Beach, a massive embayment developed immediately downdrift of the armour stone breakwater. This embayment started to develop as a small indentation in 1985 and expanded until November 1986. Field research by LaValle (1985–1995) indicated that a combination of successively high lake levels and the presence of the armour stone breakwater immediately north of the Northeast Beach could have generated the necessary conditions to scour the embayment. This paper focuses on assessing the development of this embayment for the period 1985–1995, and examines whether the various growth stages of the embayment can be fitted to logarithmic-spiral curves.

DATA ACQUISITION

Since May 1978, a team from the University of Windsor, in cooperation with Parks Canada, has been engaged in a shoreline monitoring programme at the Northeast Beach, Point Pelee. The major objectives of the programme are to map the topography and bathymetry of the Northeast Beach semi-annually, and to analyse the spatial and temporal changes occurring to the beach and shoreline. In the process of data acquisition at the Northeast

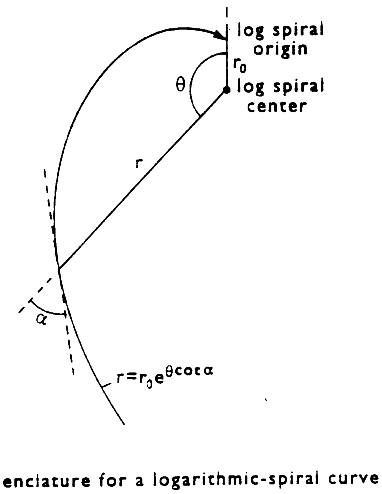


Figure 2. Nomenclature for a logarithmic-spiral curve (after Terpstra and Chrzastowski, 1992, p. 604).

Beach, a large shallow embayment was observed in May 1985. This embayment started to expand progressively. A spatial assessment of this phenomenon was, therefore, initiated. To assess the growth of the embayment since its inception in May 1985, the shoreline position of the northern sector of the Northeast Beach was surveyed at 20 m intervals twice a year, starting in 1985. An automatic level was used to trace out the configuration of the embayment, and the perimeters of the successive stages of the embayment system were surveyed at 5° intervals. The plan view of each recorded stage of the embayment evolution was then mapped. The acquired data permitted fitting logarithmic-spiral curves to the 21 embayment configurations measured at the Northeast Beach from May 1985 to May 1995.

FITTING THE LOG-SPIRAL CURVE

While it was Yasso (1965) who first concluded that the planforms of many headland-bay beaches followed a logarithmic-spiral, it should be mentioned that natural headland-bay planforms have been referred to by several other names, among them zeta-form (Halligan, 1906), parabolic (Mashima, 1961), crenulate (Silvester, 1970) and hook-shaped (LeBlond, 1972). The nature of the logarithmic-spiral curve has been described by several researchers, among them Thompson (1942), Lockwood (1961) and LeBlond (1979).

In brief, Thompson (1942, p. 790) stated that the two most important constants in an equiangular or a logarithmic-spiral are: (1) the magnitude of the angle of the spiral, or ‘constant angle’ and (2) the rate of increase of the radius vector for any given angle of revolution. It is well established that the angle between a radius vector to the curve at any point and a tangent to the curve at that point is always a constant. The equation for a logarithmic-spiral is generally given in polar coordinates, with the radius r being a function of the angle θ. The usual form of the equation expressed by Terpstra and Chrzastowski (1992) is:

$$r = r_0 e^{\theta \cot \alpha}$$

where r is a radius from the log-spiral centre to any point on the curve, r_0 is the radius from the log-spiral centre to the log-spiral origin, θ is the angle between r_0 and r, measured from the log-spiral centre, and α is the spiral angle (which is constant for any curve) defined as the angle between the radius vector and the tangent to the curve at any point (see Figure 2).

A key step in fitting the logarithmic-spiral curve involves locating the centre of the log-spiral curve, or the pivot or turning point, which serves as the origin for the radii (r_i) going to the shoreline perimeter. This pivot point or log-spiral centre, initially determined in the May 1985 survey, is located in the first major downdrift gap

in the tetrapod line where the first small embayment was observed. This initial pivot point is tested using procedures similar to the graphical method described by Terpstra and Chrzastowski (1992). The radii (r_i) are determined by the equation:

$$r_i = (x^2 + y^2)^{1/2}$$

where x = north–south coordinates with the pivot point at the origin, and y = the east–west coordinates with the pivot point as the origin for the coordinate system. Next, the angles θ to the sample points on the bay shoreline are determined and put into radian form. The natural logarithms of the radii (r_i) are then calculated. These natural logarithms of the radii are regressed against the angles θ yielding the regression equation:

$$1\mathrm{n}(r_i) = 1\mathrm{n}(r_0) + k\theta$$

where $k = \cot\alpha$. Taking the antilog yields the form:

$$r_i = r_0\, \mathrm{e}^{k\theta}$$

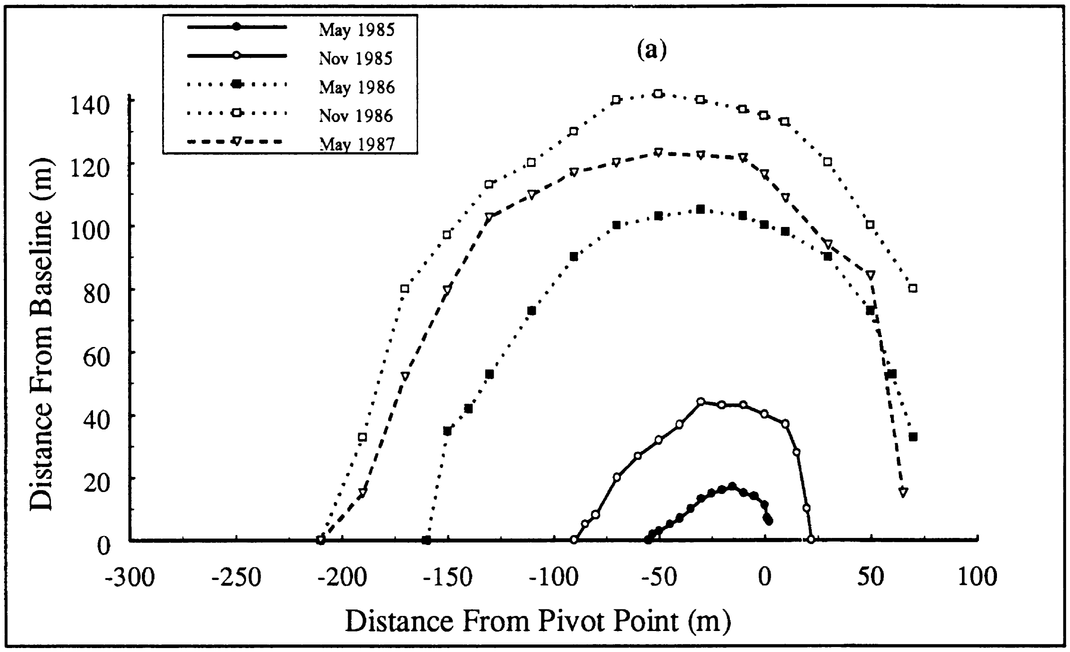
Next the error terms of this fit are tested for homogeneity of variance and spatial autocorrelation to assess the appropriateness of the model.

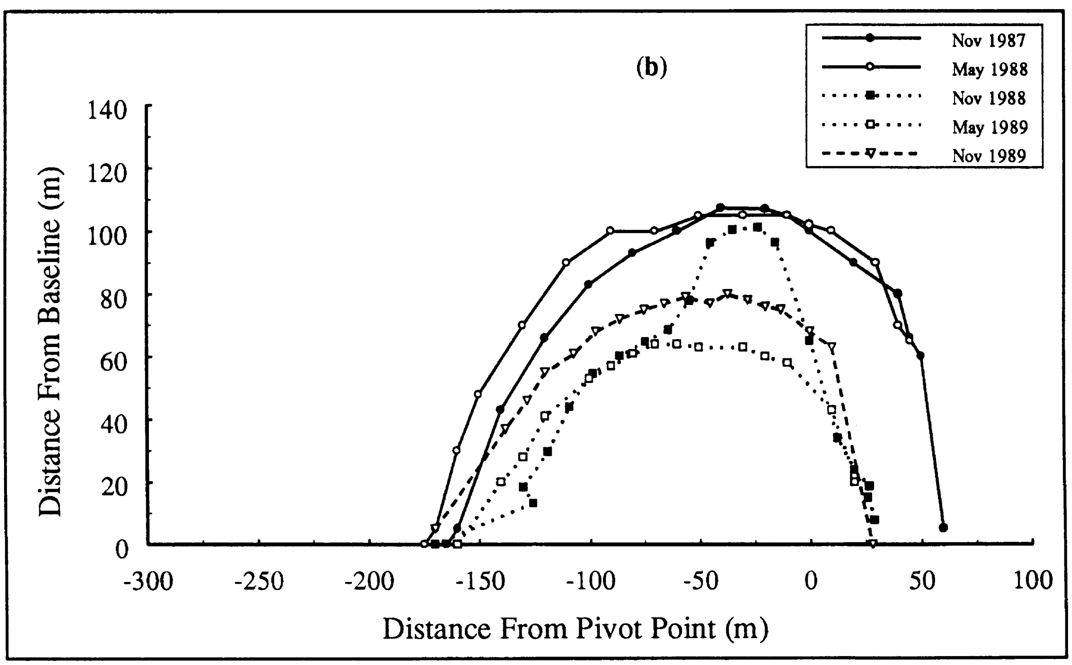
If no significant autocorrelation or heteroschedacity of variance in the residuals or error terms is detected the researcher may assume that the model is appropriate, and that the pivot point is accurately located. The presence of significant autocorrelation would mean that the pivot point has been mislocated or that there is an absence of a log-spiral system. The final step requires calculating the coefficient of determination to assess the goodness-of-fit, and if the model is to be accepted the correlations should exceed 0·98.

RESULTS

The embayment that developed along the northern third of the Northeast Beach for the period May 1985 to May 1995 has been assessed for the characteristics of log-spiral morphology. The log-spiral fits for the 21 surveys are illustrated by Figure 3a–d, while Table I presents the significant parameters and statistics associated with each log-spiral curve. In all cases the log-spiral models provide more than adequate fits to the embayment configurations. No significant heteroschedacity of variance is present among the residuals, and only one significant autocorrelation can be observed in Table I. This occurred in the November 1988 survey after a period of relatively low water levels and can be associated with the emergence of the line of tetrapods. The emergence of the tetrapods can be blamed for disrupting the processes associated with the wave diffraction and refraction patterns governing the growth and maintenance of the log-spiral embayment system. The appropriateness of the log-spiral fits is further demonstrated by the coefficients of determination which, with the exception of that for the November 1988 dataset, are all over the required value of 0·98 (see Table I). The coefficients of determination attest to the fact that log-spiral modelling is appropriate for describing the crenulate bay forms observed at the Northeast Beach between 1985 and 1995. It should, nevertheless, be pointed out that distortions in the log-spiral fit can be introduced by artificial structures which interfere with shoreline processes.

The computed regression coefficients, k, show a range between 0·28 for May 1986 to 1·41 for November 1990. All of the regression coefficients are significantly greater than 0·0 at the 0·05 level. Terpstra and Chrzastowski (1992) have shown that these regression coefficients represent the cotangent of the angle, α, which is the angle between any radius vector and the tangent to the shore curve at the point where it meets the radius vector. Figure 4 illustrates the behaviour of k over time. It is evident that k drops to relatively low values during the period of rapid embayment growth, especially from November 1985 to November 1986. This occurrence supports the research of Wind (1994) who implied that in the expanding phase of a log-spiral system, growth is faster in the diffraction zone. After November 1986, water levels dropped to a low in 1988 and the k values rise to a peak of 1·41. During the 1988–1990 period the embayment experienced a period of


(a)
May 1985
Nov 1985
May 1986
Nov 1986
May 1987
Distance From Baseline (m)
Distance From Pivot Point (m)
0
20
40
60
80
100
120
140
-300
-250
-200
-150
-100
-50
0
50
100


(b)
Nov 1987
May 1988
Nov 1988
May 1989
Nov 1989
Distance From Baseline (m)
Distance From Pivot Point (m)
0
20
40
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-200
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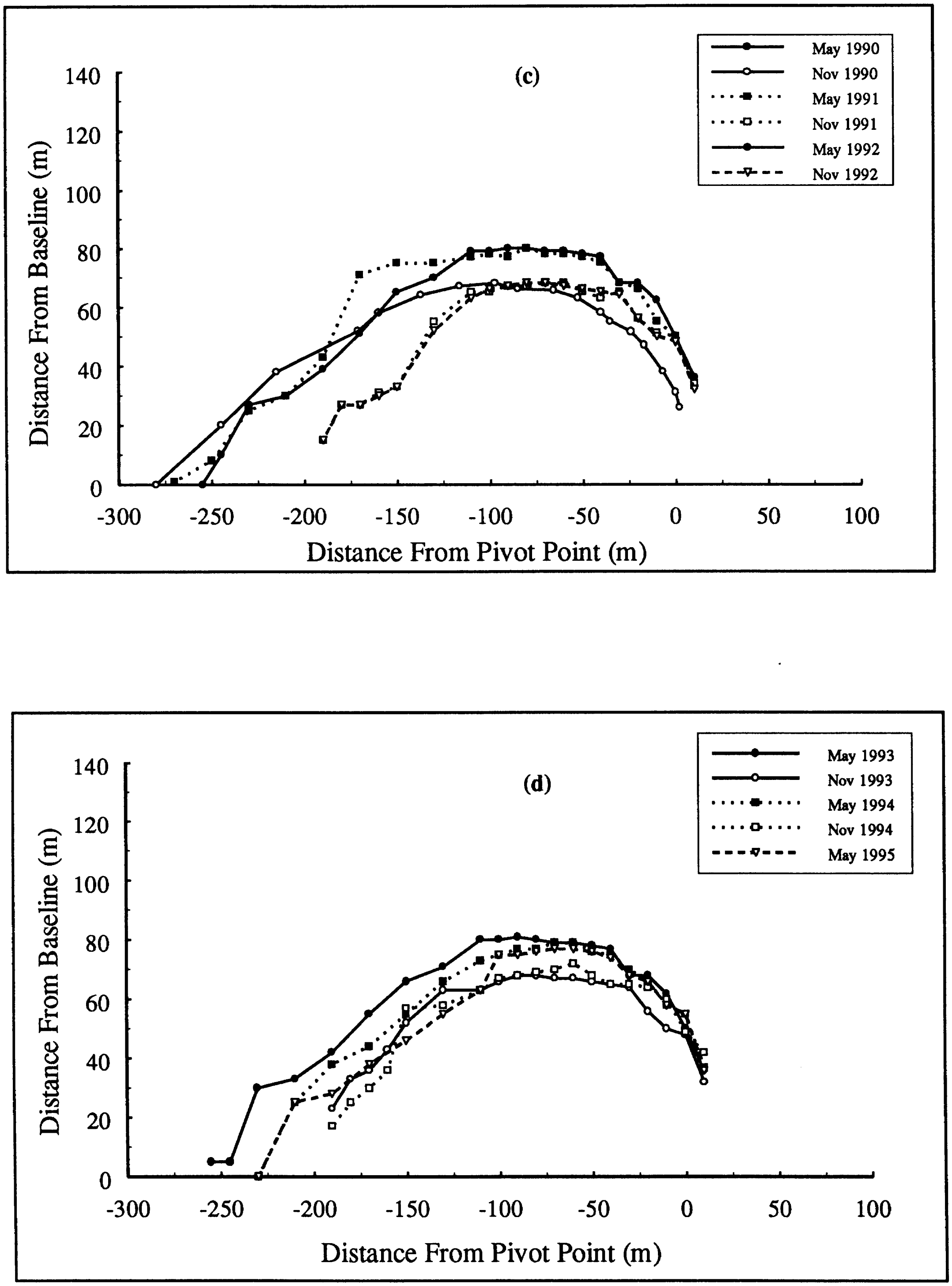

Figure 3. (a), (b) Northeast Beach log-spiral embayment, May 1985 to November 1989. (c), (d) Northeast Beach log-spiral embayment, May 1990 to May 1995.

Table I. Summary of logarithmic-spiral regression analyses for the log-spiral bay evolution

Survey period	Regression equation[a]	t-test Ho: k=0[b]	R^2	Residual autocorrelations Lag=1	Lag=2	Lag=3	Bartlett's χ^2[c]
May 85	R=1·71e$^{1·09a}$	35·4*	0·990	0·15	–0·14	0·02	3·16
Nov 85	R=0·90e$^{0·45a}$	26·9*	0·981	0·09	–0·05	0·12	2·55
May 86	R=0·95e$^{0·28a}$	45·9*	0·993	0·23	–0·00	0·09	2·46
Nov 86	R=1·00e$^{0·29a}$	36·1*	0·989	0·25	–0·13	–0·16	1·74
May 87	R=0·91e$^{0·37a}$	40·2*	0·991	–0·10	–0·10	–0·21	3·42
Nov 87	R=1·00e$^{0·32a}$	45·9*	0·993	0·23	0·06	–0·08	4·02
May 88	R=1·16e$^{0·35a}$	36·8*	0·989	0·20	–0·30	–0·32	3·35
Nov 88	R=0·85e$^{0·60a}$	11·5*	0·939	0·71*	0·27	–0·11	4·92
May 89	R=0·90e$^{0·73a}$	115·0*	0·999	0·26	–0·30	–0·37	3·35
Nov 89	R=28·22e$^{0·56a}$	66·6*	0·998	–0·15	0·14	0·13	0·68
May 90	R=26·58e$^{0·58a}$	89·7*	0·998	0·00	0·29	0·05	2·60
Nov 90	R=3·28e$^{1·41a}$	169·6*	0·999	–0·34	–0·09	–0·07	4·17
May 91	R=9·00e$^{1·07a}$	68·2*	0·996	–0·04	–0·07	–0·40	1·10
Nov 91	R=10·51e$^{0·94a}$	48·2*	0·992	0·03	–0·33	0·06	3·73
May 92	R=10·46e$^{1·01a}$	75·2*	0·998	0·16	–0·14	–0·07	6·14
Nov 92	R=10·07e$^{0·95a}$	57·1*	0·994	0·13	–0·30	–0·16	4·34
May 93	R=10·30e$^{1·02a}$	80·5*	0·997	–0·06	–0·17	–0·02	4·69
Nov 93	R=9·09e$^{1·01a}$	60·5*	0·995	0·11	–0·30	0·15	5·07
May 94	R=11·67e$^{0·95a}$	74·0*	0·997	–0·23	0·20	–0·11	5·95
Nov 94	R=12·42e$^{0·89a}$	50·2*	0·993	0·15	–0·03	0·03	4·37
May 95	R=12·04e$^{0·93a}$	45·9*	0·991	0·23	0·21	–0·22	0·89

[a] R=predicted radius; a=angle in radians
[b] *=Significant at the 0·05 level.
[c] For d.f.=3, χ^2 critical at the 0·05 level=7·81

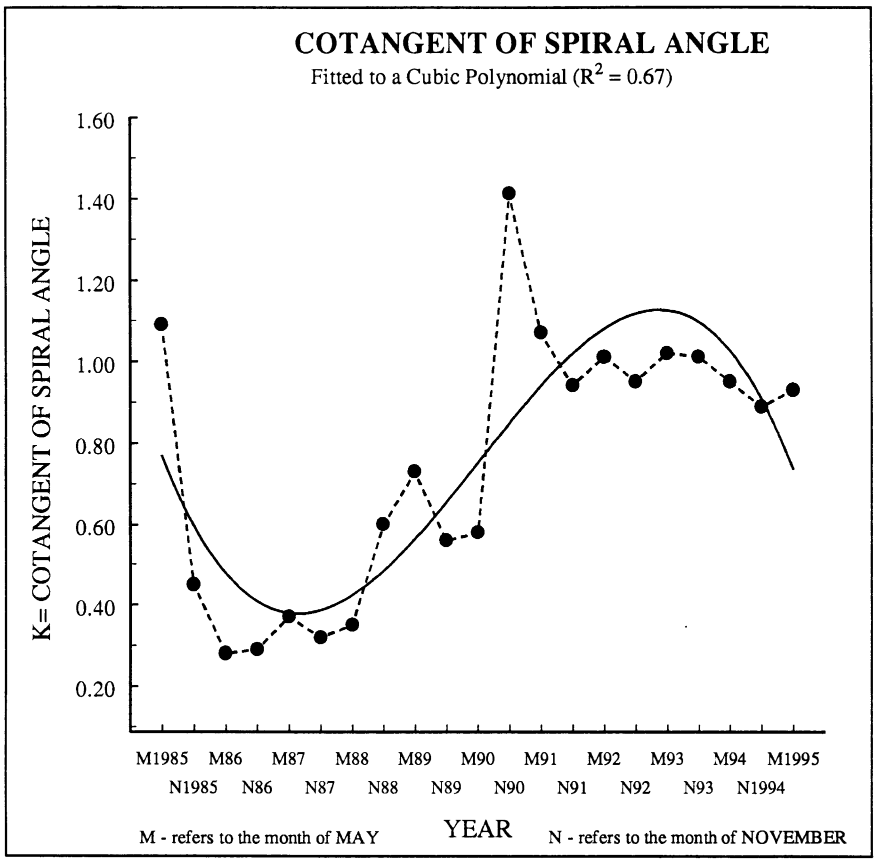

Figure 4. Temporal trends in the cotangent of the spiral angle.

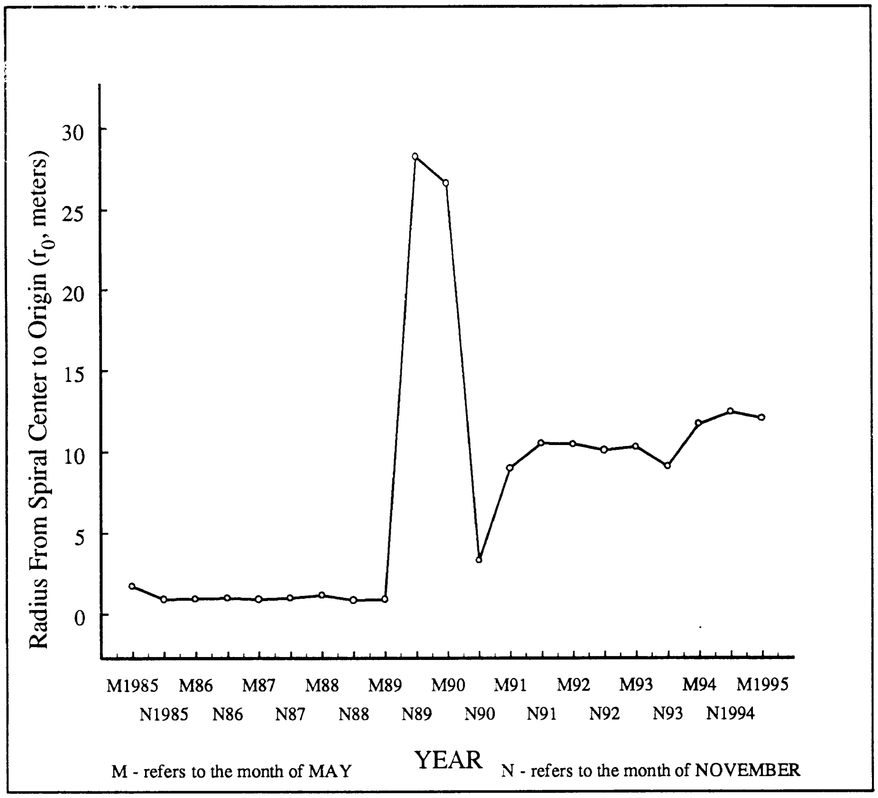

Figure 5. Temporal trends in the radius (r_0) from the log-spiral centre to the log-spiral origin.

significant readjustment. Since 1991, the embayment k values display relatively stable characteristics in the 0·90–1·02 range.

Figure 5 illustrates the r_0 values, the antilogarithms of the regression intercepts representing the spiral radii for angles, $\theta = 0$ over time. These values are quite small during the genesis of the log-spiral embayment, then they rise suddenly to the 24–28 m marks about a year after the emergence of the tetrapods during low water. This indicates a progressive readjustment process in the system. After an increase in water levels to the 174·8–174·9 m marks, the r_0 values levelled off around the 10 m mark, indicating the attainment of a new quasi-equilibrium state in the system.

The maximum landward penetration of the embayment over time is illustrated by Figure 6. It can be observed that there is a rapid change from 15 m to over 140 m between May 1985 and November 1986. With a drop in water levels, the log-spiral shaped embayment migrated lakeward and remained until 1989. After 1989, the landward penetration of the spiral embayment appears to have stabilized between 65 and 75 m from the original 1984 shoreline position.

DISCUSSION

The findings from the Northeast Beach are similar to those of other investigators (e.g. Silvester, 1974; Komar, 1983) who reported that constructed jetties, groynes, harbours, breakwaters and other artificial structures can lead to occurrences of severe beach and shoreline erosion. Silvester (1974) clearly pointed out that artificial structures are less effective in the long term because they shift erosional problems to other coastal areas. The embayment which developed at the Northeast Beach can be attributed to the scouring forces generated by the long armour stone breakwater immediately north of this scoured beach area. If properly emplaced, the line of

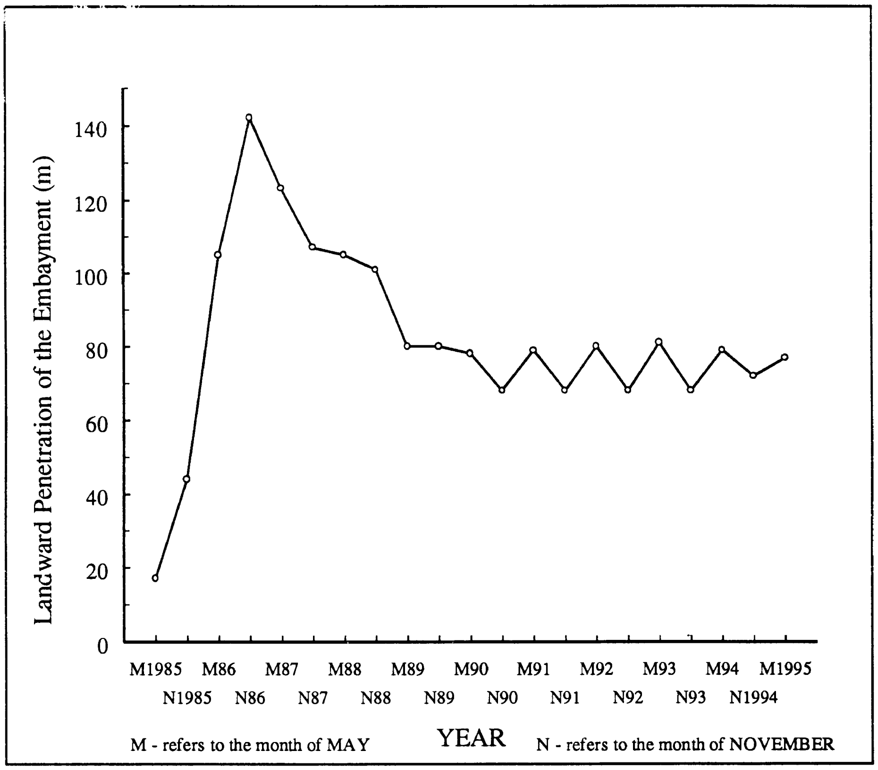

Figure 6. Temporal trends in the landward penetration of the embayment.

tetrapods extending over a 260 m distance at the Northeast Beach should have prevented the scouring from occurring. However, in the vicinity of profile 210, the tetrapod subsided and a 15 m gap allowed wave and current scouring forces to initiate the growth of the embayment. As lake levels rose to record heights, the scouring forces intensified in this gap and fanned out to scour the embayment. At its peak, this embayment attained a length of nearly 300 m, and can be associated with the more than 140 m of shoreline retreat which occurred between 1985 and 1986. As the embayment expanded progressively, the log-spiral morphology became evident. The logarithmic-spiral form, in its incipient and subsequent stages, is verified by the statistical results presented in Table I.

The drop in water levels to a low of 174·3 m in November 1989, resulted in a reduction in the size of the embayment. During this period, the log-spiral shaped embayment readjusted to different hydrodynamic conditions. However, between November 1990 and May 1995 mean water levels rose to between 174·8 and 174·9 m above mean lake level. This occurrence allowed the log-spiral shaped embayment to gain some stability between 1991 and 1995. The initial seasonal fluctuations and later stable form of the embayment are clearly exhibited by the plotted log-spiral parameters. Both the spiral angle, α (Figure 4), and the radius, r_0 (Figure 5), show decreases through time, with only very minor fluctuations after May 1991. These results suggest that the embayment is shifting away from its initial quasi-equilibrium condition, and could be approaching a state of dynamic equilibrium. While it is difficult to conclude that ‘a developing log-spiral planform is in a state of dynamic equilibrium from the earliest stage of development and as the form expands landward and downcoast’ (Terpstra and Chrzastowski, 1992, p. 615), this study, nevertheless, provides results which clearly show that the embayment, in all its stages of development, maintained a stable log-spiral configuration.

This persistence of the logarithmic-spiral form, irrespective of the rapid changes in lake levels and the naturally occurring variance in wave and sediment dynamics, is indicative of an equilibrium shape configuration. Supporting evidence from the literature (e.g. Walton, 1977; Hsu *et al.*, 1993) show that the

logarithmic-spiral form denotes the equilibrium embayment shape that might be expected on the flank of artificial structures. Moreover, the pioneering study by Yasso (1965) also postulated that well fitted log-spiral curves are representative of an equilibrium configuration. Given the progressive adjustment of the Northeast Beach embayment to more stable log-spiral characteristics, a configuration resembling a static equilibrium bay (see Hsu *et al*., 1989, 1993) could eventually be attained.

CONCLUSION

This study has shown that, in spite of the presence of a triple line of concrete tetrapods to control erosion, the presence of the long armour stone breakwater may have initiated the evolution of the embayment in the northern sector of the Northeast Beach. The growth and development of this embayment intensified because of rising lake levels. Although through time the size of the scoured embayment adjusted to reflect shoreline hydrodynamic and sedimentologic conditions, the evolved configuration, nevertheless, matched at all times the logarithmic-spiral form. The close agreement of the successive stages of embayment developed to the logarithmic-spiral form throughout the study period allows the conclusion to be made that the embayment may be approaching a dynamic equilibrium state. Since coastal engineers and shoreline managers are interested in logarithmic-spiral configurations because they could stabilize the shoreline (Silvester and Ho, 1972; Suh and Dalrymple, 1987) it would be worthwhile to investigate further the evolution and persistence of logarithmic-spiral forms in order to develop quantitative relationships and analytical models for the stability of log-spiral planforms. If it could be ascertained that there is a distinct statistical relationship between the persistence of a log-spiral planform and shoreline stability, then coastal engineers and resource managers could incorporate the log-spiral concept into shoreline protection strategies.

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